2020

MATHEMATICS — HONOURS

Sixth Paper

(Module - XI)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

Group - A

[Vector Calculus - II]

(Marks : 10)

1. Answer ***any one*** question :

(a) Verify Stoke's theorem for a vector field defined by $\vec{F} = \left(x^2 - y^2\right)\hat{i} + 2xy\,\hat{j}$ in the rectangular region in *xy*-plane bounded by the straight lines $x = 0,\ x = 5,\ y = 0,\ y = 8$; 10

(b) Prove that for any scalar function φ (*x*, *y*, z),

$$\iiint \vec{\nabla}\varphi\, dv = \iint \varphi\, \hat{n}\, dS$$

where $\hat{n}$ is the outward drawn unit normal vector to the surface *S*. 10

(c) If *V* is the region bounded by the planes *x* = 0, *y* = 0, *z* = 0 and 2*x* + 2*y* + *z* = 4, then show that

(i) $\iiint\limits_V \vec{\nabla}\times\vec{F}\, dV = -\frac{8}{3}\hat{k}$

(ii) $\iiint\limits_V \vec{\nabla}\cdot\vec{F}\, dV = \frac{16}{3}$ where $\vec{F} = \left(3x^2 - 8z\right)\hat{i} - 2xy\,\hat{j} - 8x\,\hat{k}$ 6+4

(d) Verify Green's theorem for the line integral $\oint\limits_C \left(x^2 + xy\right)dx + x dy$, where *C* is the bounding curve of the region traced by $y = x^2$ & $y = x$. 10

Group - B

[Analytical Statics - II]

(Marks : 20)

Answer ***question no.*** **2** and ***any one*** question from the rest.

2. (a) Find the centre of gravity of the arc of the parabola $y^2 = 16x$ included between the lines $x = 0$ and $x = 4$. 6

Or,

(b) Find the condition of stability of equilibrium of a mechanical system having one degree of freedom. 6

3. A solid frustum of a paraboloid of revolution of height h unit and latus rectum 8 unit rests with its vertex on the vertex of a paraboloid of revolution whose latus rectum is 4 unit. Show that the equilibrium is stable if $h < 2$. 14

4. Forces $\vec{X}, 2\vec{X}, 3\vec{X}$ act along the vectors $\hat{i}+\hat{j}-\hat{k}$, $\hat{i}-\hat{j}+\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively. Find the resultant wrench, pitch and intensity. 14

5. A force $\vec{P}$ acts along the axis of x and another force $n\vec{P}$, where n is a positive integer, acts along a generator of the cylinder $x^2 + y^2 = a^2$. Show that the central axis lies on the cylinder

$$n^2(nx - z)^2 + \left(1 + n^2\right)^2 y^2 = n^4 a^2.$$ 14

6. State and establish the principle of virtual work for a system of co-planar forces acting on a rigid body. 14

Group - C

[Analytical Dynamics of a Particle - II]

(Marks : 20)

Answer ***question no.*** **7** and ***any one*** question from the rest.

7. (a) A particle moves with central acceleration μ/r^3. Where r is the distance of particle from centre of force. If it be projected from an apse at a distance 'a' from the centre of force with a velocity equal to $\sqrt{2}$ times that in a circle, find the path. 4

Or,

(b) Classify the equilibrium point for the linear system $AX = \dot{X}$, where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\dot{X} = \dfrac{dX}{dt}$, for different values of the scalars a, b, c and d. 4

8. A particle of mass M is at rest and begins to move under the action of a constant force $\vec{F}$ in a fixed direction. It encounters the resistance of a stream of fine dust moving in the opposite direction with velocity $\vec{u}$, which deposits matter on it at a constant rate σ. Show that its mass will be m, when it has travelled a distance $\frac{k}{\sigma^2}\left[m - M\left\{1+\log\left(\frac{m}{M}\right)\right\}\right]$ where $k = F - \sigma u$.

[F and u are the magnitudes of the force $\vec{F}$ and the velocity $\vec{u}$ respectively]. 16

9. A small bead starts sliding down a semicircular wire of radius *'a'* with coefficient of friction μ. If it starts with a velocity *'u'* from one extreme point in the upper end, find the time taken to slide down to the lowest point (Assume that the wire is fixed in a horizontal base with its centre upwards and the diameter of the free ends is horizontal). Also find the increased velocity at that point. 10+6

10. A particle describes an ellipse under inverse square law about a focus. If it is projected with a velocity of magnitude V from a point at a distance l from the centre of force, find the periodic time. 16

11. Determine the eigenvalues and corresponding eigenvectors of the following linear dynamical system :

$$\frac{dx}{dt} = 2x + y$$
$$\frac{dy}{dt} = x + 2y$$

Classify its equilibrium points. 6+6+4